H. P. HALL and R. G. FULKS

The use of active devices in precision bridges

Active devices make possible the design of impedance-measuring instruments with important new features by performing such operations as isolation, phase inversion, and integration. The disadvantages accompanying such devices are, in the main, overcome by the use of multistage transistor-feedback amplifiers. Several measuring circuits, including a new incremental inductance bridge, are discussed

Active devices have been used for many years in impedance-measuring instruments. In impedance meters, such as the Q meter or capacitance meter, the active devices are vacuum-tube voltmeters used to indicate some quantity, and the accuracy of the device is limited to that of the indicating meter. Active devices are generally used in the null-detecting system of precision bridge measurement systems, but there the active elements are not in the actual calibrated measurement circuit, and variations in the parameters of the active elements do not affect the over-all accuracy. More recently, linear active circuits have been placed directly in bridge circuits in order to achieve operating characteristics otherwise unattainable. This article discusses the advantages and limitations of this use of active devices.

BRIDGE CIRCUITS USING ACTIVE DEVICES

The use of active devices permits a wide variety of measurement circuit configurations, of which only a very few have yet been described in the literature. These complete measurement systems all use only a few types of active devices, each of which performs a specific circuit function.

Simple applications of each type of device will first be considered, and then more complex systems will be described. For this discussion, the devices will be considered to be ideal; subsequently their practical limitations will be discussed.

Isolator Applications • Perhaps the simplest, most precise, and most useful active element is the unity-gain amplifier or isolator. The conventional cathode follower (or emitter follower) is an imperfect example of this device, which ideally has infinite input impedance, zero output impedance, and a voltage gain of plus unity.

One of its most useful applications is its ability to form an adjustable, low-impedance voltage source as shown in the simple transformer bridge circuit of Fig. 1. Here the potentiometer is not loaded by the standard capacitor C_S, and the output resistance of the potentiometer is not placed in series with C_S. This circuit is particularly useful when a continuous bridge-balancing adjustment is required and a variable standard element is impractical. A variable capacitance divider could replace the potentiometer in high-frequency applications.

A second application uses the "Miller Effect,"[1] commonly associated with vacuum-tube grid-to-plate capacitance multiplication where the voltage gain K is negative. In the circuit of Fig. 2, the gain is positive and adjustable from 0 to 1, thus producing an effective variable input capacitance that goes to zero. This capacitance could be used as a variable capacitor in a bridge arm. A modification of this application would be to put the potentiometer in an adjacent arm (as in Fig. 3) so that only one isolator is necessary. Another application using this Miller effect reduction in capacitance is the guard circuit for 3-terminal measurements shown in Fig. 4. Here the two "stray" capacitances C_A and C_B have no effect on the bridge balance conditions, since C_A is effectively multiplied by zero and C_B is placed between the isolator output and ground.

It should be noted that all of these functions can also be performed by the "dual" of the unity-voltage-gain device which is a unity-current-gain device having zero input impedance and infinite output impedance. Also, an isolator can be made with a gain greater than unity, which is sometimes necessary, but such a device is generally less stable.

Inverter Applications • A phase inverter, or negative-gain amplifier, is often required to produce a null balance. Unfortunately, it is rather difficult to make a practical circuit that approaches the characteristic of an ideal voltage amplifier with a gain of minus unity. Perhaps the most practical approximation is the shunt feedback amplifier shown in Fig. 5. This has a finite input impedance, but if a high input impedance is necessary it could be preceded by an isolator. The dual of this circuit, shown in Fig. 6,

Essentially full text of paper no. 62-290, presented at the AIEE Winter General Meeting, New York, N. Y., Jan. 28-Feb. 2, 1962.
H. P. Hall and R. G. Fulks are with the General Radio Company, West Concord, Mass.

has infinite input impedance but a finite output impedance, and it could be followed by an isolator to get a low output impedance.

A simple inverter application, shown in Fig. 7, is a bridge suitable for comparing impedance where stray impedance from either unknown terminal to ground has no effect, because it shunts the generator or detector. Since the voltage gain is dependent upon the value of the two resistors, either of them theoretically could be used for a balancing adjustment.

Integrator and Differentiating Application • The two impedances in the inverter described above (or its dual) need not be resistors but could both be complex impedances. The two most obvious cases would be to replace either R_1 or R_2 (of Fig. 5) with a capacitor, thereby forming a differentiator or integrator and obtaining either a leading or lagging 90-degree voltage phase shift. If a 90-degree phase shifter replaces an inverter in the circuit of Fig. 7, a resistor can be used to balance an unknown reactance, or vice versa.

A more general way to look at this type of circuit is to realize that the relationship between the four impedances of the circuit of Fig. 7 is the same as the relationship between the four impedances in a 4-arm bridge. Thus, any of the conventional bridge circuits as, for example, the Maxwell bridge of Fig. 8, can be redrawn with the use of this new configuration. This last circuit was considered for an incremental inductance bridge since direct current could be fed to the unknown through chokes in parallel with input and output, and the impedance of these chokes would not affect the null conditions.

Voltage-to-Current and Current-to-Voltage Converter Applications • The inverter circuits of Figs. 5 and 6 may be considered as an ideal current-to-voltage and a voltage-to-current converter, respectively, with resistors added. If R_1 is removed from the circuit of Fig. 5, the remaining device has, ideally, a zero input and output impedance and a transfer resistance of R_2. Likewise, if R_2 is removed from the circuit of Fig. 6, this circuit has infinite impedances and a transconductance of $1/R_1$. A simple application of this device is in the measuring circuit of Fig. 9, which requires fewer components than the very similar circuit of Fig. 7.

Other Devices • Another useful active device is the differential amplifier that has been used in place of an output bridge transformer.[2] This device must be considered as part of the bridge as well as part of the detector, since it directly affects the null conditions if its common-mode rejection is not infinite. It has the advantages over a bridge transformer of placing less stray capacitance across the arms of the bridge and being less subject to magnetic pickup. There are so many references to differential amplifiers that its design will not be considered here.

Another active device which conceivably may be used to advantage would be a high-gain amplifier to

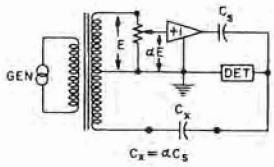

Fig. 1. Bridge using an adjustable voltage source

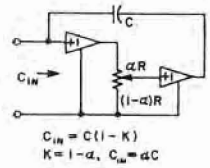

Fig. 2. Variable capacitance multiplication

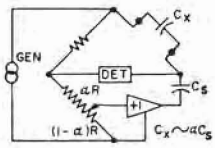

Fig. 3. Bridge using capacitance multiplication

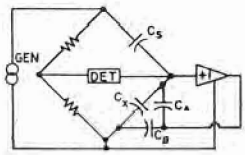

Fig. 4. Active guard circuit

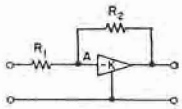

Fig. 5. Parallel feedback inverter

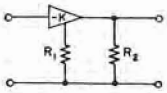

Fig. 6. Series feedback inverter

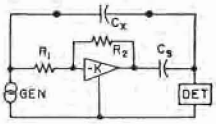

Fig. 7. Bridge using inverter

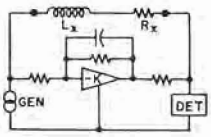

Fig. 8. Active Maxwell bridge

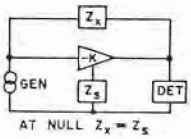

Fig. 9. Bridge using voltage-to-current converter

provide a high-voltage ratio, but great care and many active elements would be required to get a high-gain device with the same long-term stability as that of a unity- or low-gain device.

Such devices as impedance inverters[3] and converters[4] could conceivably also be used to advantage, but it is hard to imagine a measurement problem which could not be more easily solved with several of the simpler devices.

Instruments Using Active Devices • There are a few references to impedance measurement circuits which use one or more of the active devices described above. Konigsberg[5] describes what he calls an operational bridge for measuring capacitors up to 30 μf in the 0.05-to-10 cps range. Essentially, he compares the outputs of two active differentiators, one using the standard capacitor and the other the unknown. He gets 0.25 per cent accuracy on this bridge and suggests a modification that would measure inductance.

A mutual inductance bridge is described by Pil-

linger, Jastram and Daunt[6] in which an adjustable voltage is transformed by means of a vacuum tube into a proportional, adjustable current, which is used to drive a fixed mutual inductor. This arrangement avoids the need of a variable mutual inductor previously used for similar measurements.

An a-c 4-point-probe resistivity bridge devised by Logan,[7] shown in general form in Fig. 10, uses isolators and an inverter. This circuit makes possible measurements on a 4-terminal unknown without a preliminary balance, such as required by a Kelvin bridge. It can be easily shown that, if $Z_1 = Z_2$, then $Z_X = \alpha RZ_1/Z_3$. Note that all active devices used in this circuit have a common ground.

Integrators have been used in commercial transfer-ratio bridges to obtain the quadrature voltage reference required. Where complex transmission characteristics are measured by comparison with a simulated circuit, it is difficult to decide whether the device is a bridge or an analog computer. This latter field has a vast literature on the design and use of operational amplifiers.

A New Incremental Inductance Bridge • A simplified circuit diagram of a new incremental inductance bridge is shown in Fig. 11. It uses two isolators and a negative voltage-to-current converter. The null conditions can be easily obtained by setting $I_1 + I_2 + I_3 + I_4 = 0$. If we let R_B equal the total resistance to ground from point B, including R_f, then

$$\frac{I_1 + I_2 = I_3 + I_4}{E_{\text{in}}} = \frac{R'_B}{R'_B + R_x + j\omega L_x}\left(\frac{1}{R_f} + \alpha j\omega\, C_s + \alpha\beta G_s\right) - \frac{R_d}{R_e\,(R_c + R_d)} = 0 \qquad (1)$$

or

$$\frac{1}{R_f} + \alpha\, j\omega C_s + \alpha\,\beta\, G_s = \frac{R'_B + R_x + j\omega L_x}{R'_B} \times \frac{R_d}{R_e\,(R_c + R_d)} \qquad (2)$$

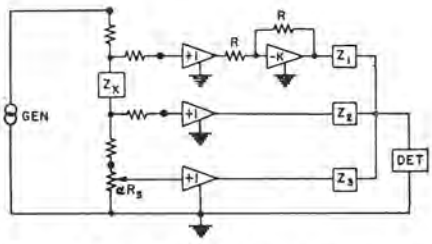

Fig. 10. Logan's bridge for 4-terminal measurements

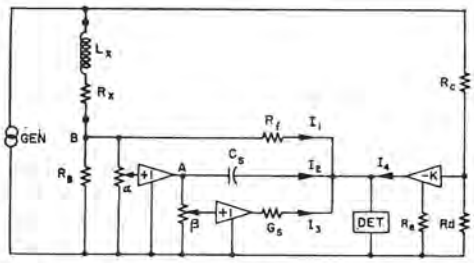

Fig. 11. An incremental inductance bridge

If

$$\frac{1}{R_f} = \frac{R_d}{R_e\,(R_c + R_d)}$$

then

$$L_x = \frac{\alpha\, C_s\, R'_B\, R_e\,(R_c + R_d)}{R_d} \qquad (3)$$

$$R_x = \frac{\alpha\,\beta\, G_s\, R'_B\, R_e\,(R_c + R_d)}{R_d} \qquad (4)$$

$$Q_x = \frac{\omega\, L_x}{R_x} = \frac{\omega\, C_s}{\beta\, G_s} \qquad (5)$$

The potentiometer adjustment α can be calibrated to read L_X and β to read Q_X. If the second potentiometer is connected to point A instead of point B, it can be easily shown that β will be proportional to R_X instead of $1/Q_X$.

The advantages of this circuit are

1. It will read either Q_X or R_X. Use of the R_X connection when low-Q inductors are measured avoids the "sliding null" (slow balance convergence).
2. The bridge can easily be made to read Q directly at any number of frequencies if a switch is used to place the required value of G_S in the circuit.
3. The current through the unknown inductor is not affected by the balance adjustments. (A variable impedance in series with a nonlinear unknown can cause annoying difficulties.)
4. The resistor R_B may be of very low value since it is fixed and therefore the voltage applied to the unknown is very nearly equal to that applied to the bridge input terminals (a feature particularly important for measurements on nonlinear inductors).
5. Because R_B is small, a large direct current may be passed by the unknown without excessive power dissipation.
6. Because the alternating voltage across R_B is only a small fraction of the voltage across the unknown, a large voltage may be applied without overdriving the active devices.
7. The value of C_S may be changed at different frequencies to provide a reasonable inductance range at any frequency.
8. The oscillator and detector are both grounded and no output bridge transformer is used that would be subject to magnetic coupling to the unknown inductor or generator.
9. Both balance adjustments are continuously variable, making rapid adjustments possible.

This bridge has 1 per cent accuracy, limited mainly by the ability to read the dials. It is direct reading in Q at nine frequencies between 50 cps and 15.75 kc and has an over-all range from 0.1 μhenry to 1,000 henries.

PRACTICAL LIMITATIONS OF ACTIVE DEVICES

Unfortunately, the ideal active devices used in the preceding discussion are not available (and for that

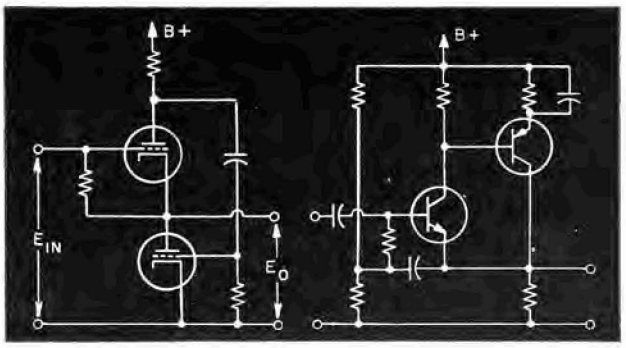

Fig. 12. (left) A White cathode follower

Fig. 13. (right) A transistor White cathode follower

matter, neither are the ideal passive components). Active elements traditionally have a rather long list of undesirable characteristics which may justifiably cause some hesitancy in their use by bridge designers. Each of these disadvantages will be discussed in order to show that the use of transistor devices with liberal amounts of feedback overcome or reduce many of them, and that others can be avoided by proper application in the measurement circuit.

Accuracy and Long-Term Stability • Each of the devices has a transfer parameter that enters directly into the null expression of any bridge it is used in. Obviously, this parameter must have a stability substantially better than the stated accuracy of the bridge.

In practical circuits, this critical parameter differs from the ideal value by an error term which could be reduced to any degree only if there were no limit on the number of active elements that could be put in the feedback loop. The loop gain is limited, however, by loop stability considerations (here stability means nonoscillatory). To make the loop unconditionally stable, the loop gain must decrease with frequency at a controlled rate until it is less than unity, and therefore the maximum loop gain at the operating frequency is related to the excess bandwidth. The residual error term can be compensated for, or accounted for in the calibration, but only to the extent of its stability. Here transistors might have the advantage over vacuum tubes of stability with time, and vacuum tubes the advantage of stability with temperature.

The archetype for the isolator circuit is the White cathode follower[8] shown in Fig. 12; this is a special case of the common feedback amplifier in which the output is tied directly to the cathode of the input stage. Properties of typical vacuum-tube and transistor versions of this device are given in Table I. The transistor circuit of Fig. 13 uses complementary elements for high d-c stability.

If the power gain of the second stage of such an amplifier were infinite, the voltage gain of the whole would depend only on the μ (open-circuit voltage gain) of the first stage and would be $\mu/(1+\mu)$. This is mentioned to bring out the point that the μ of a transistor (grounded emitter) is very large, usually 1,000 or more, which is even higher than that of most pentodes, and therefore transistor isolators can have gains very close to unity.

A third active element may be added in a variety of ways to get better performance. Two useful circuits thus formed are shown in Figs. 14 and 15. In the first, the added transistor can be considered as forming a compound "cascode" input stage, which has very high μ. In the second, an emitter follower is added in the loop to increase the loop gain. Properties of these two devices are also given in Table I. These three transistor circuits showed open-circuit gain changes of about 0.03 per cent when the supply voltage was reduced to one half its design value.

The inverter circuit of Fig. 5 is not dependent upon the μ of the first stage because currents, rather than voltages, are subtracted to detect differences in between output and input. However, its voltage gain depends on two resistors which, though precise, add two extra tolerances to the over-all accuracy expression. A 3-stage inverter was built with three high-frequency transistors to get a large and stable loop gain. It used 20-kilohm resistors for both R_1 and R_2, and the input resistance was greater than R_1 by 0.2 ohm (which is the resistance from point A to ground; see Fig. 5). The output resistance was 0.02 ohm. A measurement of the voltage gain does not indicate the transfer parameter accuracy, since it depends mainly on the accuracy of the resistors. To get an idea of the stability, the supply voltage was reduced to one half of its design value and the gain changed by only 35 ppm.

Integrators, as used in analog computers, have been widely discussed in the literature. Most of these are designed for very-low-frequency applications, require d-c stabilization techniques, and have narrow bandwidth. For most a-c bridge applications, they become much simpler devices.

The remaining devices employ the same design principles as previously described. Those requiring

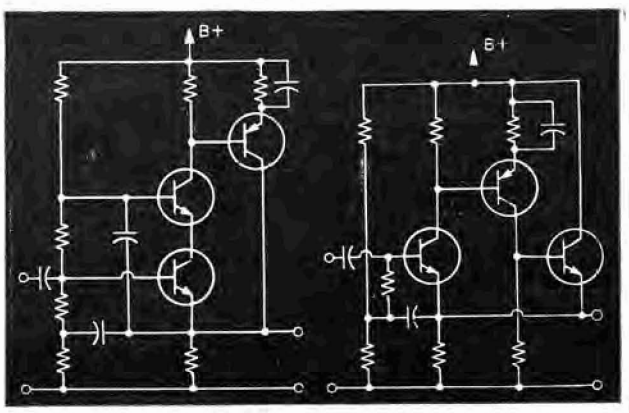

Fig. 14. (left) A White cathode follower with a cascode input stage

Fig. 15. (right) A 3-stage isolator

TABLE I • *Measured isolator characteristics (1 kc)*

Circuit	Gain		Input R, megohms		Output R, ohms	
	$R_L = \infty$	$R_L = 1$ KΩ	$R_L = \infty$	$R_L = 1$ KΩ	$R_s = 0$	$R_s = 100$ KΩ
Triode-triode (12AX7)	0.990	0.979	20	10	14	20
Pentode-triode (6U8)	0.9983	0.990	50	20	5	7
2 Transistor	0.9991	0.9983	12	8	1.8	7
3 Transistor, cascode	0.99975	0.9972	50	8	4	15
3 Transistor, high gain	0.9993	0.9992	6	6	0.05	1.0

a high output resistance have the special problem that direct current must be fed to their output stages through an impedance that does not appreciably shunt the output. An active feed circuit with a high a-c impedance is often required.

Input and Output Impedances • In many cases the input and output impedances of the device are a more important source of error than its transfer parameter, because of loading attenuation which enters the null expression. This situation is somewhat aggravated by the dependence of the input impedance upon the load and the output impedance upon the driving source impedance, as shown in Table I. This is a result of the finite power gain of any active device, and in most cases a reasonable choice of circuit impedances will avoid impossible design requirements.

Much has been written about methods of designing transistor amplifiers with very high input impedances, and it is not very difficult to get values of 100 megohms or more if the load impedance is not too low. This high impedance is accomplished by driving or guarding the necessary bias resistors from the emitter to get the same Miller effect multiplication previously described; see Fig. 13. The low internal base-to-emitter resistance is also multiplied. Thus, a transistor isolator with a gain very close to unity can have very high input impedance since the multiplying factor becomes very large; see Fig. 2. A large load on the output reduces the voltage gain and thus reduces the effectiveness of the guard. When very high input impedances are required, low grid-current vacuum tubes are generally used, particularly if the device is heavily loaded since such vacuum tubes can obtain high impedances without the guard action. Fortunately, considering the applications described here, such high impedance would rarely seem to be required.

Low output impedances below 1 ohm are often required as in the case of the inductance bridge of Fig. 11. Here, the capacitor C_S is 1μf at 120 cps, and appreciable resistance of the driving isolator would cause phase shift and hence a Q error. The circuit used was that of Fig. 15, which had an 0.05-ohm output resistance.

Noise • Active devices in a bridge circuit will introduce additional sources of noise, which may limit the balance precision when low-level signals are used. To evaluate the effect of these new sources of noise, they can be compared to other sources such as thermal noise in passive components and noise generated in the first stage of the detector.

As an example, consider the simple circuit of Fig. 1. Noise in the isolator is attenuated by the voltage divider formed by the standard and unknown components. When the standard is of much lower impedance than the unknown, this noise is transmitted practically unattenuated to the output, where it adds directly to the detector input noise. Which of these two devices contributes the most noise depends on the design of the two input stages and the impedance levels at their inputs.

Also, the noise due to the isolator may add only slightly to the noise of the resistance of the potentiometer. Transistor amplifiers can be made to give a noise figure of 3 db or better in the 1- to 10-kilohm source-resistance range[9] (a reasonable resistance level for wire-wound potentiometers). A 3-db noise figure means that the noise power level has been only doubled. Thus, the isolator has added noise, but only of an amount comparable to that of the potentiometer.

The problem of hum pickup is greatly reduced for transistor circuits as compared to vacuum-tube circuits. Not only are there no heaters, but also the alternating current levels in the power supply transformer are usually greatly reduced.

Maximum Signal Levels • In many cases, the maximum signal level that may be applied to the bridge is limited by the restricted linear range of the active elements. Passive components also have their level limitations, but there is no real comparison between the two on this point. However, in many cases, the maximum voltage on the active elements may be relatively easily limited by design, as in the case of the inductance bridge of Fig. 11. Here, an alternating voltage of 1,200 volts may be applied to the unknown inductor with only a few volts across the isolator inputs.

Power Dissipation and Warm-up Time • The power dissipated in a precision bridge using many vacuum tubes could be a serious problem because of the temperature rise and the resulting changes in the values of the passive bridge components. With transistor circuitry, it is difficult to imagine a case where this would be a problem. Also, bridges are often used for a single measurement, and the warm-up time required by vacuum tubes could be annoying.

Life—Reliability • There is some controversy concerning the actual reliability of transistors, and nothing new will be added in this discussion. However, it is generally agreed that a properly made transistor used well within its ratings has a reliability approaching that of many passive components, and substantially better than that of conventional vacuum tubes.

Frequency Range • The applications described in this article are limited to the audio range, but it would seem that similar techniques could be used up to at least 1 mc. At high frequencies it is general-

ly more difficult to design feedback circuits with large loop gain, and therefore the over-all specifications of the amplifiers would be poorer.

There would appear to be no limit at the low-frequency end except for drift effects. Konigsberg's bridge extends down to 0.05 cps with chopper-stabilized amplifiers. D-c applications appear limited, but a d-c version of Logan's bridge might be a useful device, though it would require several good d-c amplifiers.

CONCLUSIONS

The examples given show how measurement circuits with active devices can be used to avoid many of the design restrictions and operating limitations of conventional impedance bridges. Advantages include the avoidance of impractical variable components, the elimination of preliminary balances (for 4-terminal measurements) and guard balances (for 3-terminal measurements), the possibility of having D or Q direct reading at several frequencies, simplified means for applying d-c bias to the unknown, and the removal of bridge transformers. It seems apparent that many new applications would develop as bridges with special features are required.

It is difficult to state specifically the maximum accuracy possible in measurement circuits using active devices. Certainly, most of the devices described would not appreciably reduce the calibration stability of 1 per cent bridges. On the other hand, one would not expect them to be used on the most accurate bridges used for intercomparison of impedance standards. Some of these devices could be used in bridges with accuracies of 0.1 per cent or better, if loading effects are minimized by proper design. More sophisticated feedback circuits could eventually find application in 0.01 per cent bridges.

REFERENCES

1. J. M. Miller, *Bureau of Standards Scientific Paper*, no. 351, Bureau of Standards, Washington, D. C., June 11, 1919.
2. Differential Amplifier for Null Detection, W. E. Barnette, L. J. Giacoletto. *Electronics*, New York, N. Y., vol. 28, Aug. 1955, pp. 148-51.
3. Some Gyrator and Impedance Inverter Circuits, B. P. Bogart. *IRE Proceedings*, New York, N. Y., vol. 43, July 1955, pp. 783-96.
4. Transistor Negative-Impedance Converters, J. G. Linvill. Ibid., vol. 41, June 1953, pp. 725-29.
5. Operational Bridge Gages High Capacitance, R. L. Konigsberg. *Electronics*, vol. 30, Jan. 1957, pp. 175-77.
6. Electronic AC Mutual Inductance Bridge for Measuring Small Susceptibilities at Low Temperatures, W. L. Pillinger, P. S. Jastram, J. G. Daunt. *Review of Scientific Instruments*, New York, N. Y., vol. 29, Feb. 1958, p. 159.
7. An AC Bridge for Semiconductor Resistivity Measurements Using a Four-Point Probe, M. A. Logan. *Bell System Technical Journal*, New York, N. Y., vol. 40, no. 3, May 1961, pp. 885-920.
8. Thermionic Valve Amplifier Circuit Arrangement, E. L. C. White. *U. S. Patent 2,358,428*, Sept. 19, 1944.
9. A Simplified Noise Theory and Its Application to the Design of Low-Noise Amplifiers, A. E. Anderson, R. G. Fulks. IRE *Transactions*, Professional Group on Audio, vol. AU-9, no. 4, July–Aug. 1961, pp. 106-08.

Computer dates Babylonian clay tablets

Shown below are portions of three separate pieces of a Babylonian clay tablet which dates back to 183 B.C. Tablets like these are being dated by means of the astronomical information on them through use of tables produced on an IBM computer by an IBM mathematician. The technique may help provide scholars with new insights into the pre-Christian civilization era.

Courtesy Trustees of the British Museum